chapter-7

Gravitation

→ Every body in the universe attracts every other's body with a force called force of attraction

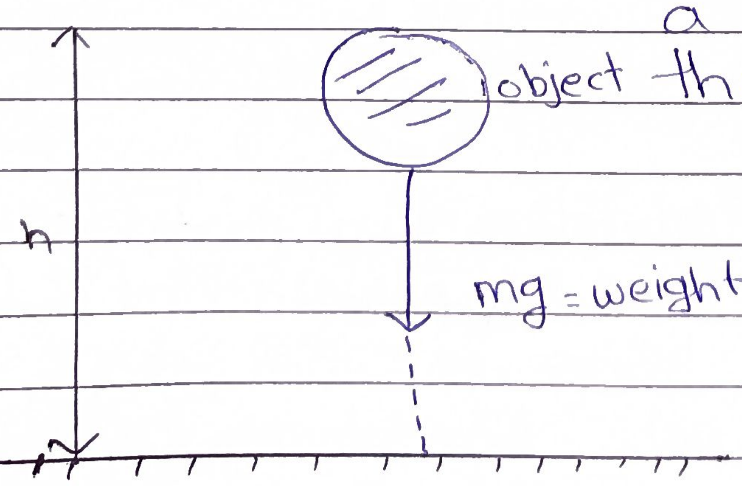
→ Gravitation is the force of attraction betn any two bodies in the universe.

Gravity : It is the force of attraction betn the earth and any object lying on or near its surface.

Freefall : The motion of a body under the influence of gravity alone is called a free fall.

Acceleration due to gravity :
The acceleration producced in a freely falling body under the gravitational pull of the earth is called acceleration due to gravity.
It is denoted by 'g' :

Weight of a body : It is defined as the gravitational force with which a body is directed towards the centre of the earth



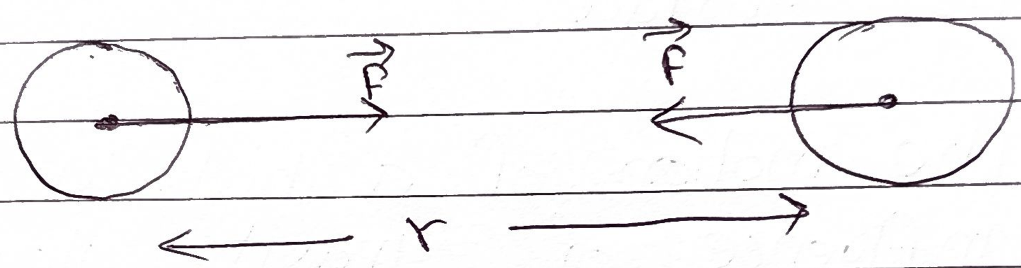
→ weight is a vector quantity. Its SI unit is Newton

$$\boxed{\vec{w} = m\vec{g}}$$

~~Netwon'~~ Newton's Law of gravitation :-

Every particle in the universe attracts every o-
particle with a force which is directly
proportional to the product of their masses and
inversely proportional to the square of distance
between them

This force acts along the line joining the two
particle



i) $F \propto (m_1 m_2)$
ii) $F \propto 1/r^2$

After combining : $F \propto \frac{m_1 m_2}{r^2}$ = $\boxed{F = \frac{G m_1 m_2}{r^2}}$

→ G is a universal constant

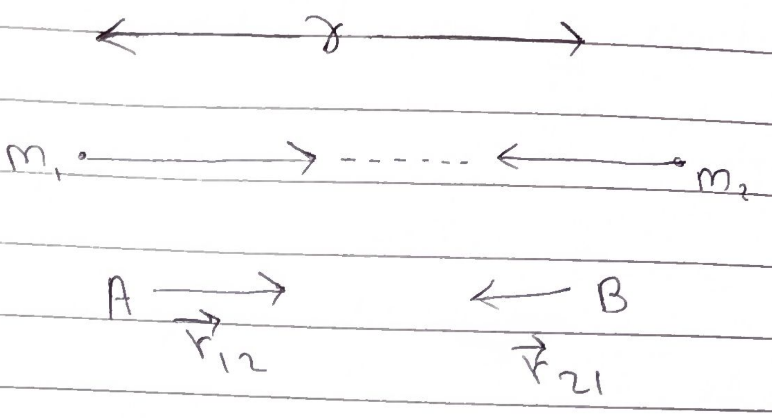
$G = 6.67 \times 10^{-11}$ Nm^2/Kg^2

Vector form of Newton's law of Gravitation

As shown in the figures, consider two particles. A
of names m_1 and m_2 and separated by
distance r.

$\vec{r}_{12}$ = a unit vector from A to B

$\vec{r}_{21}$ = a unit vector from B to A



$\vec{F}_{12}$ = Gravitational force exeerted on A by B

$\vec{F}_{21}$ = Gravitational force exerted on B by A

In vector form newton's law of gravitation can be expressed as

$$\boxed{\vec{F}_{12} = \frac{-G m_1 m_2}{r^2} \hat{r}_{21}}$$

→ The negative sign shows that the direction of force $\vec{F}_1$ is opposite to that of $\vec{F}_{21}$ that is the gravitational force is attractive in nature

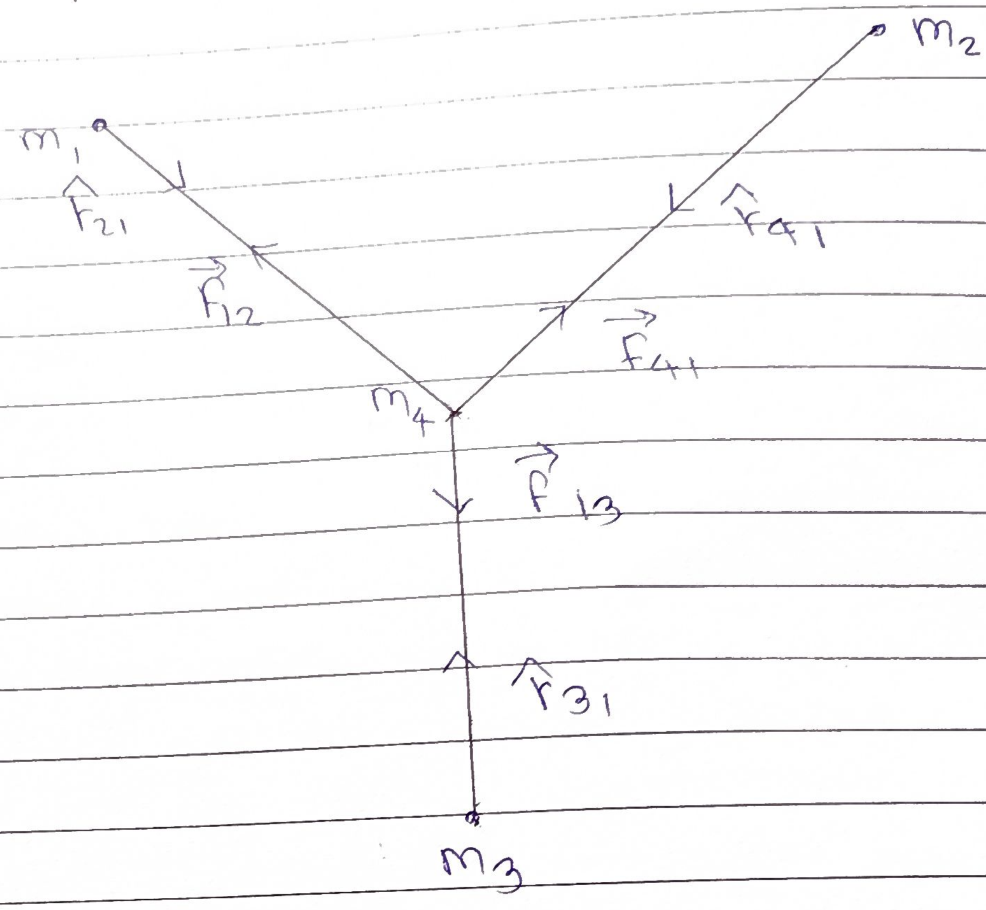
→ Similarly $$\boxed{\vec{F}_{21} = \frac{-G m_1 m_2}{r^2} - \hat{r}_{12}}$$

Important features of gravitational force :-

i) It is independent of intervening medium
ii) It obeys Newton's third law of gravitation
iii The law of gra. strictly hold for point mas
iv) The gravitational force is a conservative force.

Principle of super position of gravitational force :-

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r_{21}^2}\hat{r}_{21}$$



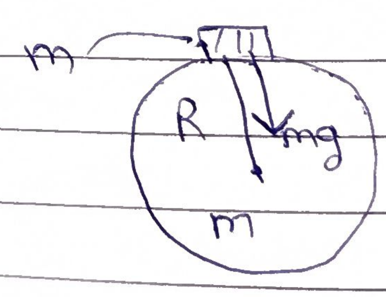
$$\vec{F}_{13} = \frac{-Gm_1m_2}{r_{31}^2}\hat{r}_{31}$$

$$\vec{F}_{14} = \frac{-Gm_1m_2}{r_{41}^2}\hat{r}_{41}$$

Total force $(\vec{F}) = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$

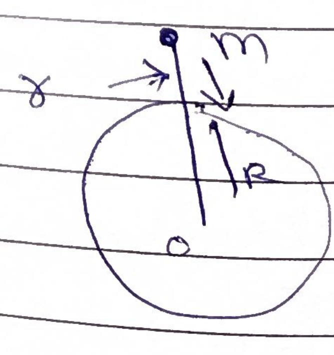
$$\Rightarrow -G\left[\frac{m_1m_2}{r_{21}^2}\hat{r}_{21} + \frac{m_3m_1}{r_{31}^2}\hat{r}_{31} + \frac{m_4m_1}{r_{41}^2}\hat{r}_{41}\right.$$

Acceleration due to gravity of the earth :-



$$F = \frac{GMm}{R^2}$$

$$F = mg$$



At equilibrium

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2} \quad \therefore g = GM/R^2$$

$$\boxed{a = g}$$

Variation in Acceleration due to gravity :-

i) Effect of altitude on g :-

$$g = \frac{Gm}{R^2} \quad \dots\dots (1)$$

$$g_h = \frac{GM}{(R+h)^2} \quad \dots\dots (2)$$

Now, $\dfrac{g_h}{g} = \dfrac{\cancel{GM}}{(R+h)^2} \times \dfrac{R^2}{\cancel{GM}}$

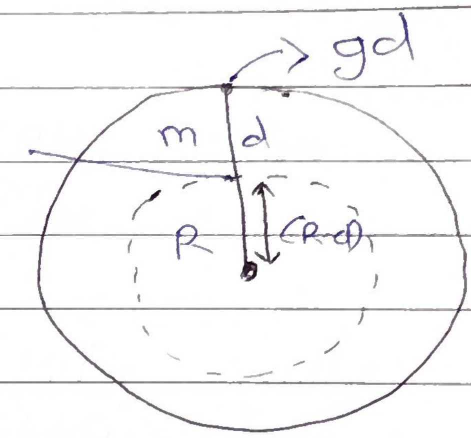
$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \boxed{g_h = g\left[\frac{R}{R+h}\right]^2}$$

$$g_h = g\left[\frac{1}{1+h/R}\right]^2 \rightarrow g_h = g\left(1+\frac{h}{R}\right)^{-2} =$$

$$g_h = g\left(1-\frac{2h}{R}\right)$$

$$\boxed{\therefore g_h = g - \frac{2gh}{R}}$$

ii) Effect of depth on g :



$$g = \frac{GM}{R^2} \quad \ldots\ldots \text{①}$$

$$g_d = \frac{GM'}{(R-d)^2} \quad \ldots\ldots \text{②}$$

$$\frac{g_d}{g} = \frac{GM}{(R-d)^2} \times \frac{R^2}{GM}$$

$$\frac{g_d}{g} = \frac{R^2}{(R-d)^2} \cdot \frac{m}{m} \quad \ldots\ldots \text{③}$$

mass = volume × density

$$m = \frac{4}{3} \pi r^3 \times \rho$$

$$= \frac{4}{3} \pi (R-d)^3 \times \rho$$

from eqn (3) we get

$$\frac{m'}{m} = \frac{(R-d)^3}{R^3}$$

$$\frac{g_d}{g} = \frac{R^2}{(R-d)^3} \times \frac{(R-d)^3}{R^3}$$

$$\rightarrow \frac{R-d}{R} = 1 - \frac{d}{R}$$

$$\boxed{\therefore \quad \frac{g_d}{g} = g\left(1 - \frac{d}{R}\right)}$$

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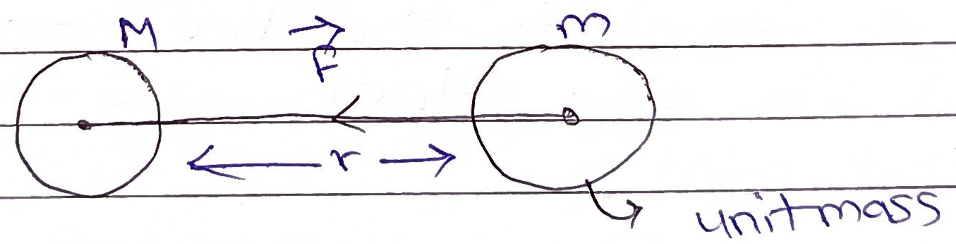
Gravitational field :

The space surrounding a material body within which its gravitational force of attraction can be experience is called its gravitational field.

Intensity of gravitational field / Gravitational field strength

It is the force experienced by a body placed at a point provided the presence of unit mass does not distroub the original gravitational field

The gra. field intensity is a vector quantity denoted by $\vec{I}$ or $\vec{E}$. It always act towards the mass produced by gra field



$$\boxed{E = \vec{F}/m \quad N/Kg}$$

Now, $F = \frac{GMm}{r^2}$

$$\frac{F}{m} = \frac{GM}{r^2}$$

$$\boxed{E = \frac{GM}{r^2} \quad \cdots (1)}$$

If the test mass (unit mass) 'm' is free to move -

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{GMm/r^2}{M}$$

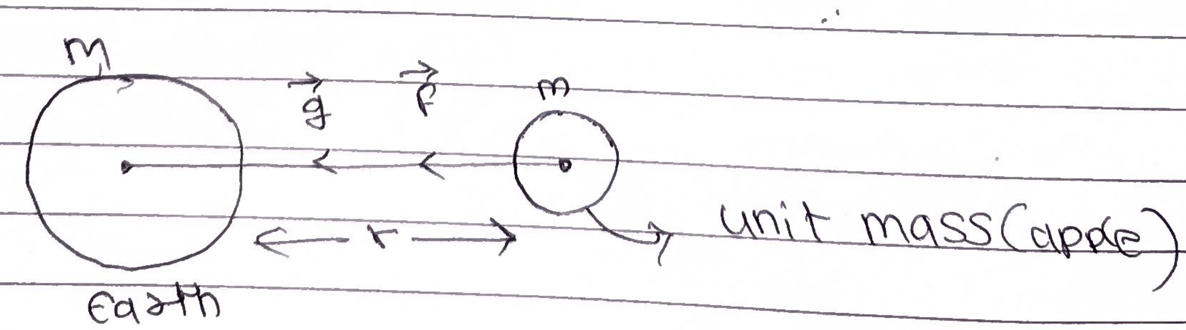
$$\boxed{a = \frac{GM}{r^2}} \quad \ldots\ldots (2)$$

It is clear that when object is free to move it can accelerate under gravitational force.

From eq^n (1) and (2) ; $\boxed{E = a}$

"Thus, the intensity of gra. field at any point is equal to the free acceleration produced in the test mass when placed at that point".

Intensity of gravitational field due to earth:



$$F = \frac{GMm}{r^2}$$

$$\left(\frac{F}{m}\right) = \frac{GM}{r^2}$$

$$\boxed{E = \frac{GM}{r^2}} \quad \ldots\ldots (1)$$

$$F = mg$$

$$g = \frac{F}{m}$$

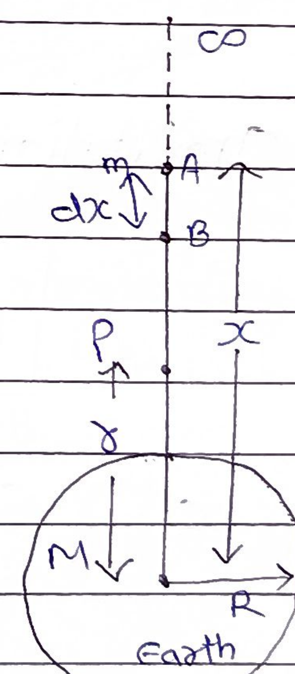
$$g = \frac{GMm/r^2}{m}$$

$$\boxed{g = \frac{GM}{r^2}} \quad \ldots (2)$$

from eqn (1) and (2)

$$\boxed{E = g}$$

Gravitational Potential Energy :-



→ The gravitational force of attraction on the body at A is :

$$\boxed{F = \frac{GMm}{x^2}}$$

→ The small workdone in moving the body through small distance dx is given by :

$$\boxed{dw = F dx}$$

$$\boxed{dw = \frac{GMm\,dx}{x^2}}$$

The total workdone in bringing the body from infinity (∞) to the point P will be :

$$W = \int_{\infty}^{r} \frac{GMm}{x^2} \cdot dx = GMm \int_{\infty}^{r} \frac{dx}{x^2}$$

$$= GMm \left[\frac{x^{-2+1}}{-2+1} \right]_{\infty}^{r} = GMm \left[\frac{x^{-1}}{-1} \right]_{\infty}^{r}$$

$$W = -GMm \left[\frac{1}{x} \right]_{\infty}^{r} = -GMm \left[\frac{1}{r} - \frac{1}{\infty} \right] = -\frac{GMm}{r}$$

$$\boxed{\therefore \; W = -\frac{GMm}{r}}$$

gravitational potential energy 'U'

$$\boxed{U = -\frac{GMm}{r}}$$

at Surface ~~tension~~ of earth

$$\boxed{U = mgh}$$

Gravitational Potential :-

The gravitational potential at a point is the potential energy associated with a unit mass due to its position in the gra. field of another body

$$\boxed{V = \frac{W}{m} \quad J/kg}$$

$$W = -\frac{GMm}{r}$$

$$\frac{W}{m} = -\frac{GM}{r}$$

At the surface of earth

$$V = \frac{W}{m}$$

$$\boxed{V = -\frac{GM}{r}}$$

Escape velocity :-

Escape velocity is the maximum velocity with which a body must be Projected vertically upward in order that it may just escaped the gra. field of the of the earth

$$\frac{1}{2} MV^2e = \frac{GMm}{r}$$

$$V^2e = \frac{2GM}{r}$$

$$\boxed{Ve = \sqrt{\frac{2GM}{r}}}$$

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All earth's surface :-

$r = R$

$V_e = \sqrt{\frac{2GM}{R}}$ $\because g = \frac{GM}{R^2}$

$gR = \frac{GM}{R}$ $\rightarrow$ $V_e = \sqrt{2gR}$

$\because g = 9.8\ m/s^2$, $R = 6400\ km$ $= 6.4 \times 10^6\ m$

$$\boxed{V_e = 11.2\ km/s}$$

Natural Satellite : A Satellite created by nature is called a natural satellite. Moon is a natural satellite of the earth.

Satellite : A satellite is a body which continuously revolve on its own around a much larger body in a stable orbit

Artificial Satellite : A manmade satellite is called an artificial satellite.

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Orbital velocity

$F_g = \frac{GMm}{r^2}$ ...... ①, $f_c = \frac{Mv^2}{r}$ .... ②

At equilibrium :-

$F_g = f_c = \frac{Mv^2}{r} = \frac{GMm}{r^2}$

$= v^2 = \frac{GM}{r} \rightarrow V_o = \sqrt{\frac{GM}{r}}$

Orbital velocity is the velocity required to put the satellite into its orbit around the earth.

$\boxed{V_1 = \sqrt{Gm}}$

Let $g = \frac{GM}{R^2}$ and $h = 0 \therefore V_o = \sqrt{gR} = \boxed{7.92\,km/s}$

Relation between orbital velocity and escape velocity :-

$\because V_e = \sqrt{2gR}$

$\boxed{V_e = \sqrt{2}V_o}$

$V_o = \sqrt{gR}$

Time period of a satellite :-

It is the time taken by a satellite to complete one ~~r~~ revolution around the earth

$$T = \frac{\text{~~Circ~~ Circumference of orbit}}{\text{orbital velocity}}$$

$$T = \frac{2\pi(h+R)}{V_0}$$

When the satellite revolve close to the earth ($h=0$) then,

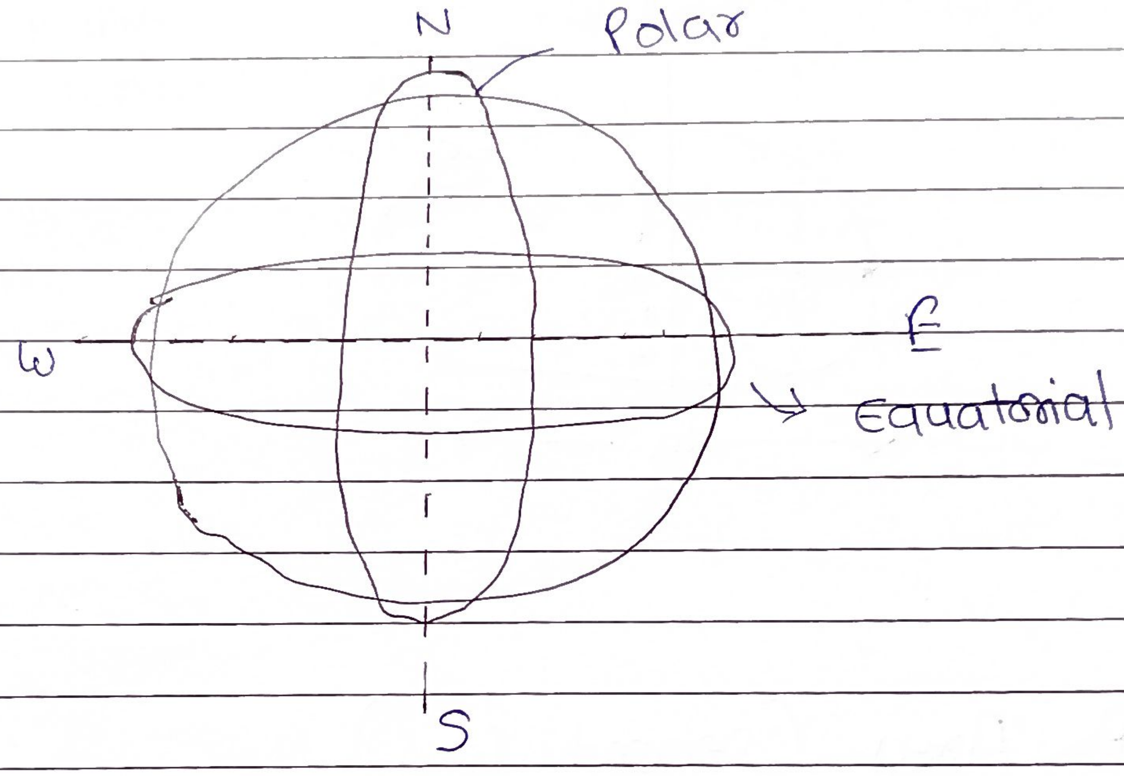
$$T = 2\pi\sqrt{\frac{R}{g}} = \boxed{84.6 \text{ min}}$$

Geo stationary satellite :-

A satellite which around the earth in its equatorial plane with the same angular speed and in the same direction as the earth rotates about its own axis is called geo stationary or synchronous satellite.

Polar satellite :-

A Satellite that revolves in a polar orbit is called a polar satellite. A polar orbit is one whose plane is perpendicular to the equatorial plane of the earth.



Uses of polar satellite :-

Polar satellites are used in weather and environment monitoring. They are used in spying for millitary purpose.

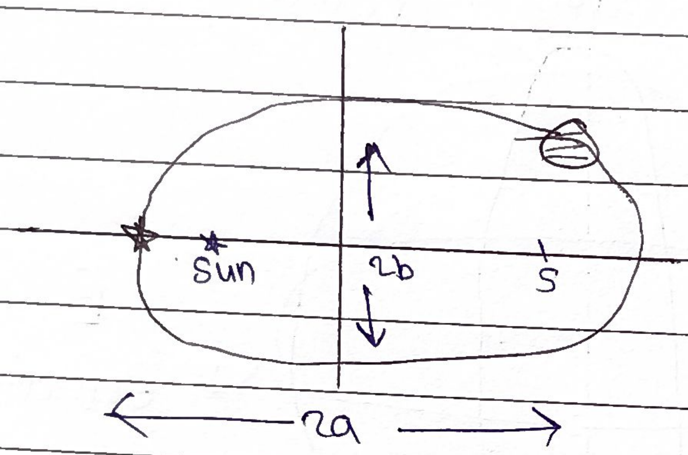
Uses of Geo- stationary satellite :-

i) In studying upper region of atmosphere

Keplar's Laws of planetary Motion :-

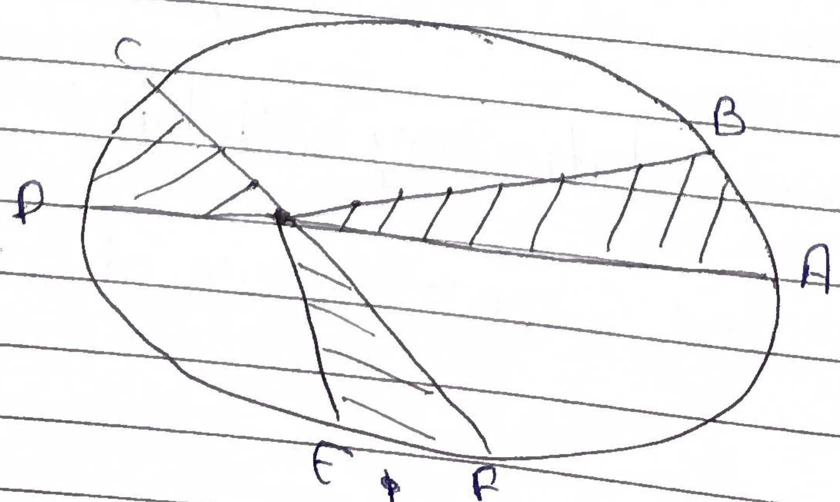
Law of orbit (first law)

Each planet revolve around the sun in an elliptical orbit with the sun situatad at one of the two focii.



Law of Area (second law)

The radius vector drawn from the sun to a planet sweeps out equal area in equal interval of time that is the a real velocity of ~~t~~ a planet around the sun is constant

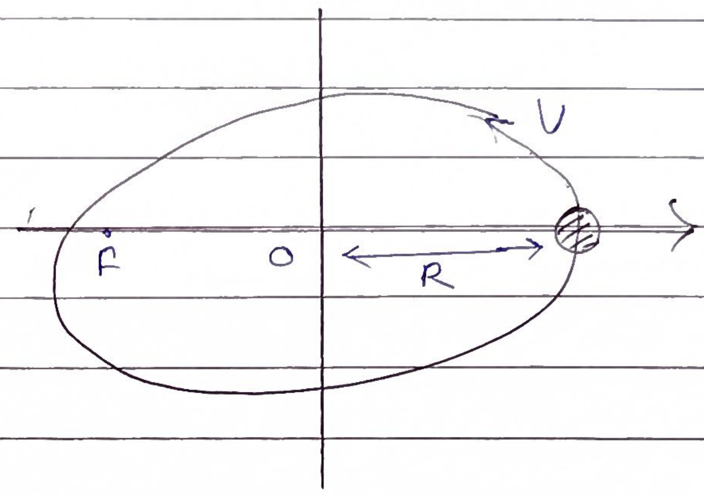


Law of Period (Third law)

The square of the period of revolution of a planet around the sun is proportional to the cube of the semi major axis of its elliptical orbit

$T^2 \propto R^3$

$T^2 = KR^3$



Weightlessness :

A body is said to be in weightlessness when ~~there~~ reaction of supporting surface is zero or its apparent weight is zero